UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

**MATHEMATICS** **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2012**

**1 hour 45 minutes**

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

*6682689949*

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.



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**1** **(i)** Prove the identity $\tan^2\theta - \sin^2\theta \equiv \tan^2\theta\sin^2\theta$. [3]

**(ii)** Use this result to explain why $\tan\theta > \sin\theta$ for $0° < \theta < 90°$. [1]

**2** Relative to an origin $O$, the position vectors of the points $A$, $B$ and $C$ are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}.$$

Find

**(i)** the unit vector in the direction of $\overrightarrow{AB}$, [3]

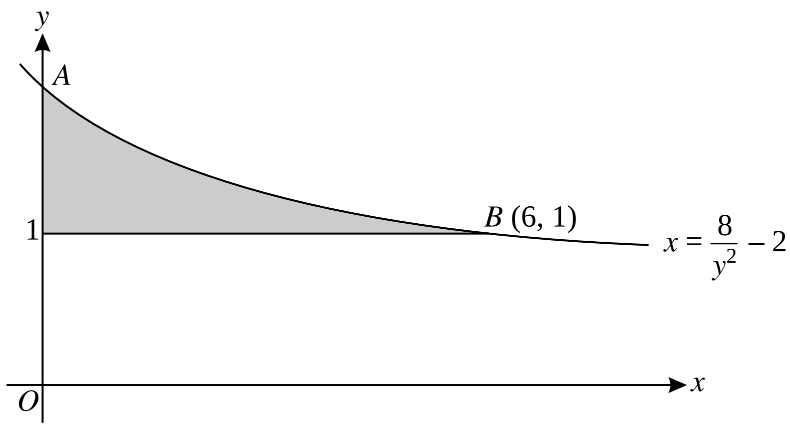
**(ii)** the value of the constant $p$ for which angle $BOC = 90°$. [2]

**3** The first three terms in the expansion of $(1 - 2x)^2(1 + ax)^6$, in ascending powers of $x$, are $1 - x + bx^2$. Find the values of the constants $a$ and $b$. [6]

**4** **(i)** Solve the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 360°$. [5]

**(ii)** How many solutions has the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 1080°$? [1]

**5**




The diagram shows part of the curve $x = \dfrac{8}{y^2} - 2$, crossing the $y$-axis at the point $A$. The point $B\,(6, 1)$ lies on the curve. The shaded region is bounded by the curve, the $y$-axis and the line $y = 1$. Find the exact volume obtained when this shaded region is rotated through 360° about the ***y*-axis**. [6]

**6** The first term of an arithmetic progression is 12 and the sum of the first 9 terms is 135.

**(i)** Find the common difference of the progression. [2]

The first term, the ninth term and the $n$th term of this arithmetic progression are the first term, the second term and the third term respectively of a geometric progression.

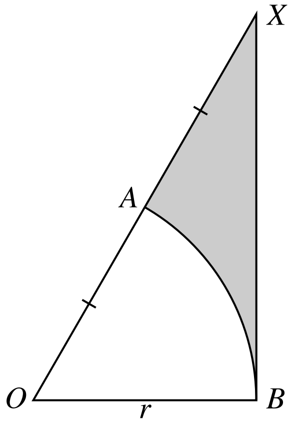
**(ii)** Find the common ratio of the geometric progression and the value of $n$. [5]

**7** The curve $y = \dfrac{10}{2x+1} - 2$ intersects the $x$-axis at $A$. The tangent to the curve at $A$ intersects the $y$-axis at $C$.

**(i)** Show that the equation of $AC$ is $5y + 4x = 8$. [5]

**(ii)** Find the distance $AC$. [2]

**8**




In the diagram, $AB$ is an arc of a circle with centre $O$ and radius $r$. The line $XB$ is a tangent to the circle at $B$ and $A$ is the mid-point of $OX$.

**(i)** Show that angle $AOB = \frac{1}{3}\pi$ radians. [2]

Express each of the following in terms of $r$, $\pi$ and $\sqrt{3}$:

**(ii)** the perimeter of the shaded region, [3]

**(iii)** the area of the shaded region. [2]

**9** A curve is such that $\dfrac{d^2y}{dx^2} = -4x$. The curve has a maximum point at $(2, 12)$.

**(i)** Find the equation of the curve. [6]

A point $P$ moves along the curve in such a way that the $x$-coordinate is increasing at 0.05 units per second.

**(ii)** Find the rate at which the $y$-coordinate is changing when $x = 3$, stating whether the $y$-coordinate is increasing or decreasing. [2]

**10** The equation of a line is $2y + x = k$, where $k$ is a constant, and the equation of a curve is $xy = 6$.

**(i)** In the case where $k = 8$, the line intersects the curve at the points $A$ and $B$. Find the equation of the perpendicular bisector of the line $AB$. [6]

**(ii)** Find the set of values of $k$ for which the line $2y + x = k$ intersects the curve $xy = 6$ at two distinct points. [3]

**11** The function f is such that $f(x) = 8 - (x - 2)^2$, for $x \in \mathbb{R}$.

**(i)** Find the coordinates and the nature of the stationary point on the curve $y = f(x)$. [3]

The function g is such that $g(x) = 8 - (x - 2)^2$, for $k \leqslant x \leqslant 4$, where $k$ is a constant.

**(ii)** State the smallest value of $k$ for which g has an inverse. [1]

For this value of $k$,

**(iii)** find an expression for $g^{-1}(x)$, [3]

**(iv)** sketch, on the same diagram, the graphs of $y = g(x)$ and $y = g^{-1}(x)$. [3]